Statistical Mechanics of Money

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1. INTRODUCTION

The application of physics-inspired thinking to economics represents an astounding interdisciplinary exercise. The subjects of economics, a soft science often lambasted for its lack of rigor, are not often thought to lend themselves to the first-principles approach to physics. However, this surprising pairing yields intellectual offspring that promise to represent even large-scale macroeconomic trends, notorious for defying prediction, to science as exact as a pure gas.

The author finds the statistical mechanics of money interesting for both of these reasons: the application of physical thinking to an unsuspecting field and the insights such an application present to one's general world view. For example, it is both reassuring and worrisome that a free market, as we shall see, produces of its own accord a certain level of inequality. However, accepting this fact encourages one to look for solutions that do not disagree with physics.

2. SCIENTIFIC ORIGIN

The scientific origin of the *econophysics* discussed in this paper begins intellectually with the analogy to the Gibbs distribution discussed in detail in the next section. The physicist Victor Yakovenko, during his undergraduate studies, took a course on statistical physics using the seminal text by Landau and Lifshitz [1]. While taking the course, he was surprised by how general the assumptions of statistical physics were, particularly in the Gibbs distribution. As he was located in the Soviet Union at the time, questions of economy were relevant; *Das Kapital* by Karl Marx was required reading for university students, for example. This inspired the young Yakovenko to apply ideas from his studies to economics, but he did not not pursue the ideas openly until he had received tenure decades later, fearing any intellectual backlash.

Other researchers, independently and through collaboration with Dr. Yakovenko, have applied physical techniques to economic ideas, too. The burgeoning field is growing. Collaborations found include researchers from North America, South America, Europe, Africa, and Asia.

New thinking that arose in the realm of economics due to this approach include attempting to create a wealth curve that represents what proportion of the population has what amount of wealth. Surprisingly, this question has not

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been attacked by classical economics to a substantial degree [1].

3. STATISTICAL MECHANICS

The statistical mechanics of money begins with an analogy to the well-known Gibbs distribution. We then consider some other theoretical consequences and analogies.

3.1. Fundamental Analogy

The fundamental analogy of econophysics is taken from the physical description of a pure gas interacting upon collisions [2]. The physical law here is derived from the rationale that the entropies of two systems must add when they are combined but that their numbers of microstates must multiply [3]. Also, crucially, the energy that is passed between interactions is conserved. Combined with the defintion of temperature T and the Boltzmann hypothesis,

$$\frac{1}{T} = (\frac{\partial S}{\partial U}), S = k_B ln(W),$$

where S is entropy, U is internal energy, k_B is Boltzmann's constant, and W is the number of microstates, we arrive at the Gibbs distribution. This distribution states that in a collection of many particles, the probability p_i that particle i has energy E_i is

$$p_i = \frac{e^{-E_i/k_B T}}{Z}$$

where Z, the partition function, is the sum over all i states. What is important here is that, as a function of energy, the probability of finding a particle in the state i decreases exponentially with increasing energy.

We now draw an analogy between the arbitrary gas described here and actors in an economy. These actors are many in number (1) and exchange energy, or "money", (2) that must be conserved (3). These three conditions suite the wealth distribution in an economy to be modelled as a Gibbs distribution. Here, the probability p_i represents (proportional to the number of actors) the number of economic actors with money M_i as

$$p_i = \frac{e^{-M_i/T_c}}{Z}$$

where T_c is the characteristic "temperature" of the system which is the average money per actor.

For the purposes of visualization, let us consider now a figure that displays this theoretical distribution of wealth. Figure 1, adapted from Yakovenko [1], shows the theoretical curve, created using a computer simulation that allowed agents to freely trade with randomized trading amounts.

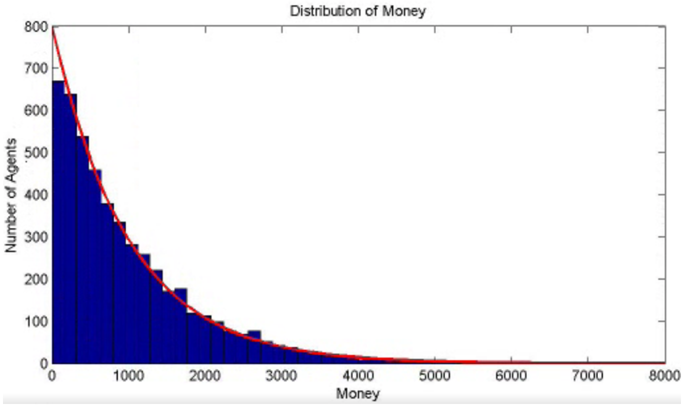

FIG. 1: Exponential distribution of wealth

3.1.1. Debt

Note that the simulation in Fig. 1 does not, evidently, permit negative money. “Negative money”, or debt, is important feature of our real economy that has gone undiscussed thus far. Economic agents may borrow money from banks or individuals, or may issue “I-O-U” tickets in order to spend money that they do not possess.

To account debt, it is useful to return to the analogy where money is the analog of energy. In physics, the problem of negative energy is usually no problem at all: the *value* of energy is arbitrary and may be defined corresponding to any reference level. The important quantity is the *difference* in energy. In the same vein, allowing a maximum debt of $-m_0$, where m_0 is positive, merely moves the vertical pseudo-asymptote from 0 to $-m_0$ while retaining the same average money, or “temperature” [2]. Mathematically, if state i has negative money $-m_i$, then the probability of occupying that state is

$$p_i = e^{-(-m_i)/T_c} = e^{+m_i/T_c}$$

where the partition function constant has been neglected for convenience. This p_i value has a positive exponential argument, implying that it is more probable than having 0 money.

Fig. 2, adapted from Yakovenko [1], shows two models, one with debt and one without debt. The model with debt was given greater average money (1800 instead of 1000) so that its plot crossed that of the debtless model.

3.1.2. Temperature

The temperature of the distribution is defined somewhat vaguely and is set at the average amount of money per agent, in part due to its success in fitting [2]. However, some economically feasible situations can produce interesting behavior in terms of temperature. Namely, a wealth cap would produce a distribution described by negative temperature.

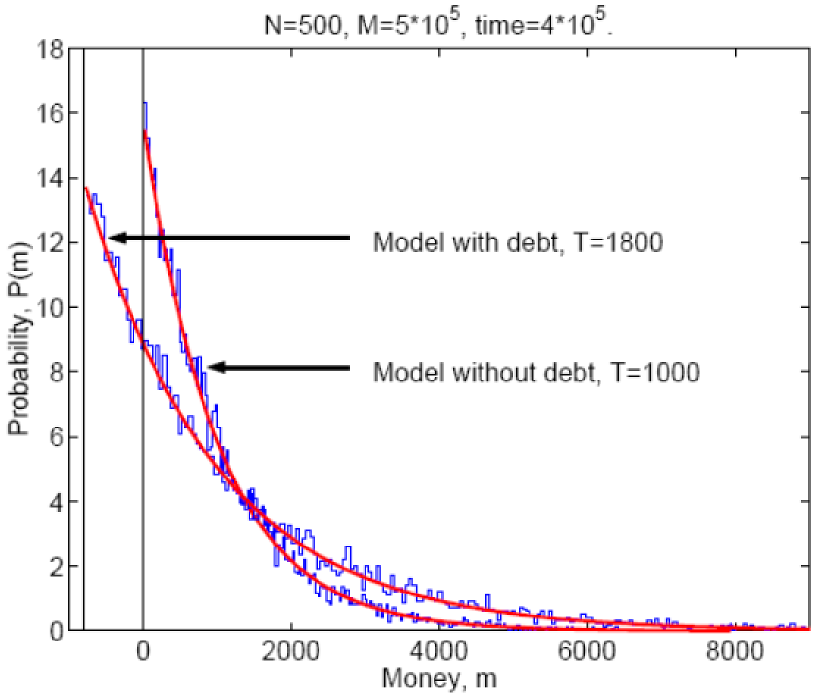

FIG. 2: Distributions with and without debt

The distribution we are now familiar with, using positive temperature, is due to a boundary condition that no agent may have negative money (or money less than a maximum debt). This leads to a large probability on the left side of the distribution. A stipulation on the *maximum* amount of money permissible will introduce a large probability on the right side of the distribution. Mathematically, this is an exponential distribution with positive arguments. This fits with the Gibbs distribution if we assign a negative temperature.

The concept of negative pressure is not unknown in physics. In laser physics, it is achieved by pumping the laser crystal to create "population inversion" where high energy states are more occupied than low energy ones, in contrast to normal materials. This is in fact the same phenomenon seen with a wealth cap. If the cap is placed at a low level, near the average money per agent, then there will be a build-up of probability in the "high-energy" state, an inversion from the usual concentration of probability at the low end of money. In an economy with both finite debt and a wealth cap, the probability distribution would be a combination of both types of exponential distributions.

3.1.3. Entropy and Inequality

Entropy is a quintessential part of any thermodynamic discussion of physical systems and the same is true here. We define entropy for a wealth distribution as

$$S = -\sum_i P_i ln(P_i)$$

where the subscript i denotes agents with money m_i. Proceeding from this definition, let us consider the entropy of an economy where all citizens have the same amount of money and there is, therefore, only one occupied money state:

$$S = -\sum_i (1) ln(1) = 0.$$

Clearly, this is the lowest entropy state and is the most unstable thermodynamically. In fact, the Gibbs distribution maximizes this entropy and is most favorable from thermodynamics. Models instantiated with perfect equality will spontaneously develop inequality as they progress to the Gibbs distribution [1]. This has profound consequences: seemingly in opposition of our societal goal to improve the finances of poor persons, we find the universe demands economic inequality with the same certainty that it compels warm ice to melt. Furthermore, the Gini coefficient

$$G = 2\int_0^1 (x-y)\,dx$$

for an exponential distribution is 1/2 [4]. This will be discussed further later in Section 4.

3.2. Experimental Data

Experimental data of wealth distributions are meager due to the difficulty of collecting accurate reports of wealth. As a substitute, income distributions, which are recorded due to taxation, are used for analysis. Figure 3, adapted from Yakovenko [5], shows the cumulative income distribution of the US in 1998 plotted on a log-log scale. On such a scale, an exponential distribution should yield a downward slope as seen up until an income of $100k. The left inset of the figure shows a log scale, where the straight line of an exponential dominates. However, the straight tail in the full-size figure is characteristic of a power law distribution [5].

Evidently, there are two empirical "classes" in the economy. A large lower class follows the exponential distribution and an upper class (the top 1-3% [5]) follows a power law distribution. We term the lower class the "thermal bulk" and the upper class the "superthermal tail" following the convention of Yakovenko [1]. We see this trend in Fig. 3, but is it true for more years? Yes, we see so in Figure 4. This figure shows the US income distributions for the 1980s and 1990s. The vertical offset between the decades has been inserted for legibility. It can be seen that all of these years are described well by a combination of exponential and power law distributions just as in 1998.

4. IMPLICATIONS BEYOND ACADEMIA

The application of physics to economics to study money distributions brings the hard science of physics into close contact with some of the questions that have the largest bearings on our lives. The first point to note to this end is how fruitful it was from a data analysis and visualization perspective to use the techniques of physics. Proper fitting of data and connection to similar mathematical models - commonplace in physics - has allowed us to better

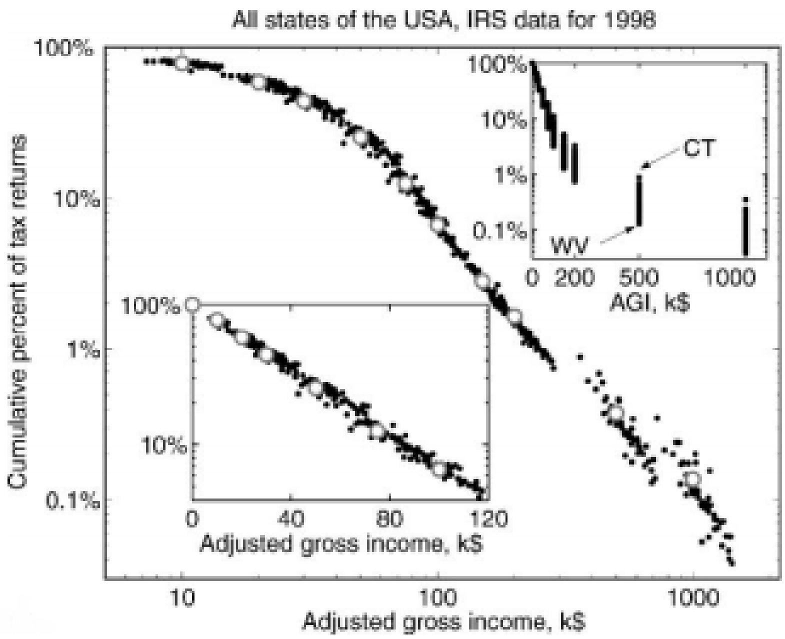

FIG. 3: The left inset is distribution on a log scale

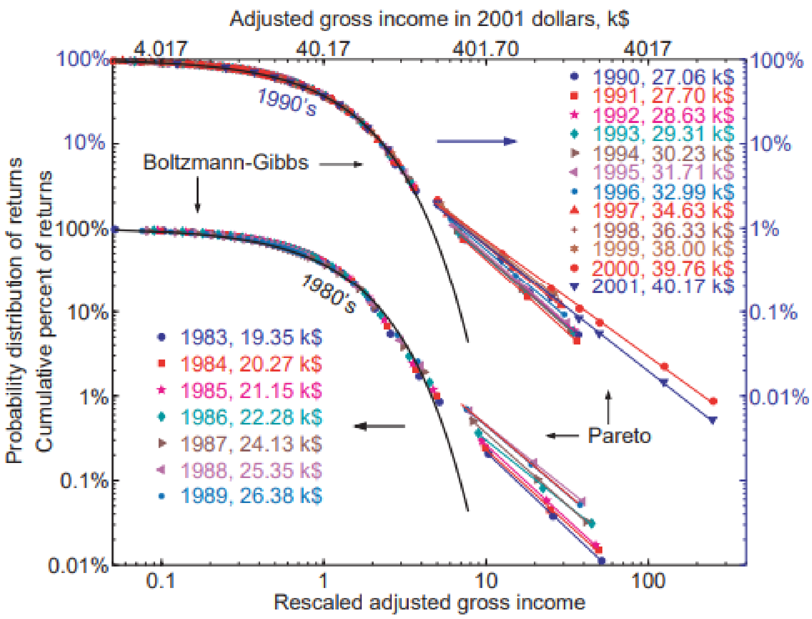

FIG. 4:

understand observations, namely the troves of IRS which had sat for decades with its large scale trends hidden.

Perhaps the most unsettling discovery in this exploration was that inequality, of the kind where many individuals are poor, is an automatic effect of free and random trading between economic agents. Indeed, it spits in the face of misguided revolutions that have tried to redistribute wealth equally since the inexorable march of entropy is guaranteed to negate the artificial equality in due time. Furthermore, this inequality translates to a Gini coefficient of 0.375 for two-earner families [4], seeming to bound how low inequality can naturally be pushed.

This newfound information may, however, be welcomed as we do our best as a society to create a just world in the face of various physical and human constraints. This analysis informs us on the physical nature of our economy so that we may better (1) characterize progress and (2) develop policies that can improve our communities while acknowledging the response of nature. The econophysics lens may soften our expectations regarding perfect economic equality between people, which undoubtedly is a blow to the most ardent champions of equity. However, the perspective shows us how far we are and suggests ways of improving wealth. The most obvious it seems is to increase the average money per agent in the economy, since increasing this "temperature" affects all points in the distribution.

Now, notice that, in Fig. 2, the model without debt was generally wealthier from its poorest agents until about 1500 units of money even though the debtless model had a lower temperature (average money). We spoke earlier about how the actual money values in a distribution were unimportant as opposed to the relative values (debt did not change the essence of the distribution). However, literal money amount is what matters to actual economic agents. A minimum wealth may be unfeasible to cultivate due to profligate spending by individuals but a minimum income may be provided by the government. Such a Universal Basic Income (UBI) would, from the econophysical perspective, act as a "negative debt" but on an income, not wealth, distribution. It should increase the income of the lower section of the distribution in a significant way. However, that added income would benefit poor agents is rather obvious. Additionally, conservation of money implies that this income would need to be taken from the present agents themselves through some taxation, likely of progressive form in order to benefit poor agents. Still, this presents a method of achieving one's goals of humane social conditions in a fashion that does not try to upend entirely the distribution of income as it is empirically found.

Another interesting artifact is the presence of the superthermal tail of the income distribution data. There is yet no clear consensus on its origin and nature. However, it has been noticed that this superthermal tail, and *not* the thermal bulk, varies in line with the stock market [1]. It is still relevant beyond academia, however, as it presents clear evidence for a two-class society, the presence and nature of which has been fiercely debated for centuries.

5. CONCLUSION

The interdisciplinary science of econophysics pairs some of humanity's best analysis tools to some of mankind's most pressing problems. To anyone natively interested in the two separately, such as the author, this burgeoning field res presents a powerful way to view the world and to judge potential solutions.

Many challenges still remain, however. Data acquisition is quite difficult since recording the data relies on strong administrative infrastructure and willingness to publicize the information which is somewhat rare even in developed countries. Furthermore, there are still theoretically uncomfortable details such as the superthermal tail.

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